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MODELING OF CYLINDRICALLY SYMMETRIC WAVE STRUCTURES IN THE OCEAN

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The problem of formation of a wave train from an axisymmetric initial disturbance caused by the fall of a celestrial body into water is solved by the method of the potential of a double layer. The main difficulty in the solution of the problem consists in calculation of two-dimensional integrals in a cylindrical system of coordinates to find the velocity potential. An algorithm that allows one to reduce calculation of two-dimensional integrals to calculation of integrals of a special form that depend on one parameter is suggested. Use of the algorithm substantially improved the efficiency of the computer program. An analysis of the results of calculation showed that two regions – of shortwave disturbances and of longwave flow – exist at the stage of formation of the wave structure from the initial disturbance.

Formulation of the Problem. A detailed description of the formulation of the problem is given in [1].

Since water is virtually incompressible, the continuity equation for it can be written in the form div $\vec{u} = 0$. We also assume that the liquid flow is nonvortical; this allows us to consider one scalar function $\vec{u} = \text{grad}\, \varphi(r, z, t)$ instead of the two components of the vector function $\vec{u}$ and one equation for the potential $\Delta\varphi = 0$ instead of two equations for the velocity components [2].

Two conditions (kinematic and dynamic) that describe the motion of the upper movable part of the boundary were considered as the boundary conditions on the upper part of the boundary. Here, continuity of pressure across the water–air interface was assumed and the action of surface tension forces was neglected. On the lower part of the boundary the condition of equality to zero of the z component of velocity was used. On the right part of the boundary the conditions of rest were formulated, i.e., it was assumed that disturbances did not reach the right-hand boundary at the considered instants of time. The conditions of symmetry were taken into account at the origin of the system of coordinates.

As initial conditions we assumed that at the initial instant of time the wave profile is known and the velocity of the liquid is zero.

Method of Numerical Solution of the Problem. The employed method of numerical solution of the above-formulated problem is a combination of the method of potentials and the method of straight lines [1]. The essence of it is the following. The nodes of the spatial grid lie only on the surface of the region. The potential of the velocity is sought in the form of the potential of a double layer, for determination of which the finite-difference approximation of the corresponding Fredholm integral equation of the second kind is used on the upper boundary. On the right-hand and lower boundaries the finite-difference approximation of the corresponding equations that are obtained by exact differentiation of the formula that approximates the velocity potential is used. In the partial differential equation for the z component of the wave profile the time derivatives are not approximated. As a result, the initial problem is reduced to the Cauchy problem for a nonlinear system consisting of ordinary differential equations and stationary equations for the values of the density function of the double layer at the nodes of the surface grid. To solve this Cauchy problem we used a STIFF package of programs modified by the present authors. This made it possible to use it in the solution of the Cauchy problem for a combined system of differential and stationary equations. The main convenience of this approach is that in the program STIFF the procedure of choosing the time step in accordance with the local accuracy of integration assigned beforehand is automated.

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Grid Construction. In construction of the grid the nodes are assigned on the wave profile and on the right-hand and lower boundaries. On the wave profile the grid is constructed in the following way. The length of the curve of the initial profile is calculated, and then this length is divided by the prescribed number of nodes. Then the nodes are equally spaced on the curve. The projections of the nodal points on the R axis form a nonuniform grid in the variable r, which is then used in the numerical simulation. This method allows one to approximate satisfactorily even almost vertical initial wave profiles (for example, 50 points are enough to approximate the initial profile of a water column whose height amounts to 5 km and whose width is only 50 m, the radius of the region of solution of the problem is 10 km). On the right-hand boundary a quasi-uniform grid in the variable z is constructed; the grid is compressed toward the water surface so that the steps in r and z at the upper right point are equal to each other. On the lower boundary a uniform grid in the variable r is used.

Approximation of the Velocity Potential. The surface profile S is a continuous curve consisting of three smooth parts: the upper boundary S_u, the right-hand boundary S_r, and the lower boundary S_b, i.e, $S = S_u + S_r + S_b$. S_r and S_b are rectilinear. The density of the potential of the double layer $\gamma(P_S)$ is a continuous, piecewise smooth function determined on the curve S. We impose a grid consisting of $N + 1$ nodes on the curve S. We approximate the curve S by a continuous broken line consisting of N links S_i, i.e., $S = \sum_i S_i$. We assume the function $\gamma(P_S)$ to be a step function on S.

Upon imposition of the grid the region of integration Ω is divided into annular segments Ω_i, with $\Omega = \sum_i \Omega_i$. Thus, to calculate the potential of the velocity we have the formula [3]

$$\varphi(M) = \sum_{i=1}^{N} \gamma(P_{S_i}) \iint_{\Omega_i} \frac{\partial}{\partial n_{P_i}} \left[\frac{1}{r_{MP_i}}\right] d\sigma_{P_i}, \tag{1}$$

where $P_i \in \Omega_i$ and $P_{S_i} \in S_i$.

The Cauchy Problem That Approximates the Initial Problem. The density of the potential of the double layer at the nodes of the grid on the surface, the z coordinate of the wave profile, and the velocity potential at the nodes of the grid on the surface are unknown quantities.

The stationary equations for the density of the potential of the double layer have the form:

a) on the upper part of the boundary

$$\varphi(M) - 2\pi\gamma(M) - \varphi^S(M) = 0, \tag{2}$$

here $\varphi^S(M)$ is the limiting values of the potential from inside, for calculation of which the corresponding equations ((5), (6), see below) should be added;

b) on the right-hand part of the boundary

$$\gamma(P_S) = 0, \tag{3}$$

c) on the lower part of the boundary

$$\frac{\partial\varphi}{\partial z} = 0. \tag{4}$$

The ordinary differential equations for the coordinate z of the wave profile and the potential on the wave surface have the form

$$\frac{dz^S}{dt} = -u_r \Lambda_r z^S + u_z, \tag{5}$$

$$\frac{d\varphi^S}{dt} = -gz^S - \frac{1}{2}(u_r)^2 - \frac{1}{2}(u_z)^2. \tag{6}$$

Here u_r, u_z are calculated as derivatives of the potential by exact differentiation of formula (1).

Thus, Eqs. (1)-(6) actually contain the values of the density function of the double layer.

Calculation of the Velocity Potential and Its Derivatives. Using the notation

$$\varphi_i(M) = \iint\limits_{\Omega_i} \frac{\partial}{\partial n_{P_i}} \left[\frac{1}{r_{MP_i}}\right] d\sigma_{P_i}, \tag{7}$$

we can write formula (1) in the form $\varphi(M) = \sum\limits_i \gamma(P_{S_i})\varphi_i(M)$. Calculating the derivative in formula (7), we have

$$\varphi_i(r, z) = 2\int\limits_0^{\pi} \int\limits_{S_i} K(r, z, \rho, \varphi, h)\, \rho\, d\vec{S}_i\, d\varphi, \tag{8}$$

where

$$K(r, z, \rho, \varphi, h) = \frac{n_r(r\cos\varphi - \rho) + n_z(z - h)}{r_{MP}^3}; \tag{9}$$

$$r_{MP} = \sqrt{r^2 + \rho^2 - 2r\rho\cos\varphi + (z - h)^2}; \tag{10}$$

the vector of the normal $\vec{n} = \{n_r, n_\varphi = 0, n_z\}$.

We introduce the notation

$$n_1 = n_r r, \quad n_2 = -n_r\rho + n_z(z - h),$$

$$c_1 = r^2 + \rho^2 + (z - h)^2, \quad c_2 = 2\rho r,$$

$$Y_\beta^\alpha = \frac{\cos^\alpha \varphi}{r_{MP}^\beta},$$

and then expressions (9), (10) are written in the form

$$K(r, z, \rho, \varphi, h) = n_1 Y_3^1 + n_2 Y_3^0, \tag{11}$$

$$r_{MP} = \sqrt{c_1 - c_2\cos\varphi}. \tag{12}$$

After exact differentiation of expression (11) we obtain formulas for calculation of the derivatives of the potential written in a new notation:

$$\frac{\partial\varphi}{\partial r} = \sum_i \gamma(P_{S_i}) \frac{\partial\varphi_i}{\partial r}, \quad \frac{\partial\varphi_i}{\partial r} = 2\int\limits_0^{\pi} \int\limits_{S_i} \frac{\partial K}{\partial r}\rho\, d\vec{S}_i\, d\varphi,$$

$$\frac{\partial K}{\partial r} = n_r Y_3^1 - 3r\, n_2 Y_5^0 - 3(n_1 r - n_2\rho)\, Y_5^1 + 3n_1\rho Y_5^2, \tag{13}$$

$$\frac{\partial\varphi}{\partial z} = \sum_i \gamma(P_{S_i}) \frac{\partial\varphi_i}{\partial z}, \quad \frac{\partial\varphi_i}{\partial z} = 2\int\limits_0^{\pi} \int\limits_{S_i} \frac{\partial K}{\partial z}\rho\, d\vec{S}_i\, d\varphi,$$

$$\frac{\partial K}{\partial z} = n_z Y_3^0 - 3(z - h)(n_1 Y_5^1 + n_2 Y_5^0). \tag{14}$$

Integrals of the form $\int\limits_{S_i} F(\rho, h)d\vec{S_i}$, where $F(\rho, h)$ is some arbitrary function, are curvilinear integrals calculated on a straight link of the broken line S. To calculate them we use the method of introduction of the parameter T. Let a link of the broken line be specified by the two points $P_1(\rho_1, h_1)$ and $P_2(\rho_2, h_2)$. We introduce the parameter T such that $T \in [0, 1]$. We determine the functions

$$\rho(T) = \rho_1 + (\rho_2 - \rho_1)T, \quad h(T) = h_1 + (h_2 - h_1)T,$$

and then the curvilinear integral is written in the form

$$\int\limits_0^1 F(\rho(T), h(T)) \sqrt{(\rho'(T))^2 + (h'(T))^2}\, dT$$

or, which is the same,

$$\int\limits_0^1 F(\rho(T), h(T)) \sqrt{(\rho_2 - \rho_1)^2 + (h_2 - h_1)^2}\, dT.$$

As a result the formulas for calculation of the potential and its derivatives acquire the final form

$$\varphi_i(r, z) = 2\int\limits_0^\pi \int\limits_0^1 K(r, z, \rho(T), h(T))\rho(T) \sqrt{(\rho_2 - \rho_1)^2 + (h_2 - h_1)^2}\, dTd\varphi, \tag{15}$$

$$\frac{\partial\varphi_i(r, z)}{\partial r} = 2\int\limits_0^\pi \int\limits_0^1 \frac{\partial K(r, z, \rho(T), h(T))}{\partial r}\rho(T) \sqrt{(\rho_2 - \rho_1)^2 + (h_2 - h_1)^2}\, dTd\varphi, \tag{16}$$

$$\frac{\partial\varphi_i(r, z)}{\partial z} = 2\int\limits_0^\pi \int\limits_0^1 \frac{\partial K(r, z, \rho(T), h(T))}{\partial z}\rho(T) \sqrt{(\rho_2 - \rho_1)^2 + (h_2 - h_1)^2}\, dTd\varphi. \tag{17}$$

The method of averages is used in numerical determination of the integrals.

Special Features of Calculation of the Derivatives of the Potential of the Double Layer. The equations of the initial problem involve the components of the velocity u_r and u_z calculated at the points P_S. This means the following:

$$u_r(P_S) = \lim_{M \to P_S} \frac{\partial\varphi(M)}{\partial r}, \quad u_z(P_S) = \lim_{M \to P_S} \frac{\partial\varphi(M)}{\partial z}. \tag{18}$$

The point M lies inside the region and approaches P_S along the normal. Since in our problem the density of the potential of the double layer is approximated by a step function, calculations by formulas (18) are not expedient because the accuracy of the result of calculations will be beyond the limits of the required accuracy of the solution of the problem. The components of the velocity u_r and u_z are suggested to be found by "receding a step inside the region," when u_r and u_z are calculated at points P_S^* obtained in receding from the point P_S by a constant value ΔL^* (the step of receding) along the inner normal:

$$u_r(P_S) \cong \frac{\partial\varphi(P_S^*)}{\partial r}, \quad u_z(P_S) \cong \frac{\partial\varphi(P_S^*)}{\partial z}. \tag{19}$$

By meaning, the value of ΔL^* should not exceed the length L of the nearest link of the broken line (L_{S_i}). The value of ΔL^* is suggested to be found by the formula

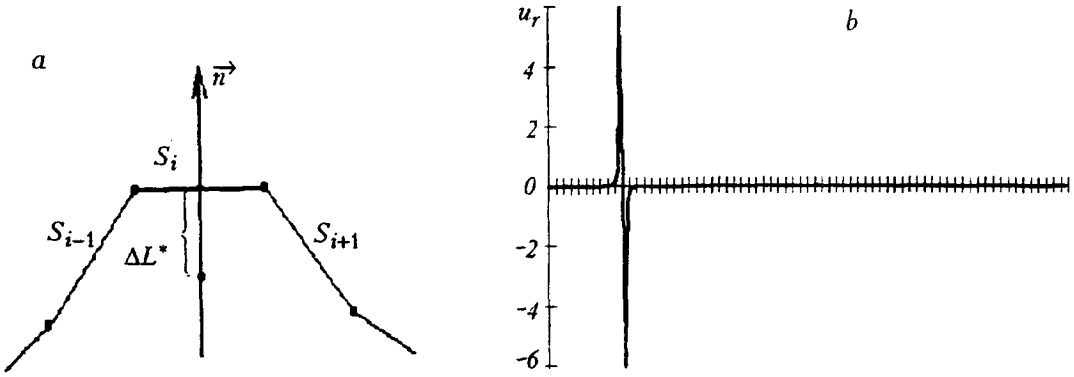

Fig. 1. Illustration of the "method of receding by a step inside the region."

$$\Delta L^* = l^* \min_i \Delta L_{S_i}, \tag{20}$$

where l^* is the adjustable parameter, $l^* > 0$; ΔL_{S_i} is the length of the i-th link of the broken line (see Fig. 1a).

The corner nodes of the right-hand boundary of the region determine the upper permissible value of the parameter l^*: $l^* < 1/2$. The smaller the step of receding, the closer, seemingly, the values calculated by formulas (19) to their exact values. However, a numerical experiment showed that for the parameter l there also exists a minimum permissible value, which is determined by the required accuracy of calculation of the derivatives of the potential and the accuracy of calculation of the integrand. We consider, as an example, calculation of the r component of the velocity for the following problem. Let us have an equilibrium undisturbed surface of liquid with a density of the potential of the double layer $\gamma = 1$. We calculate the r component of the velocity for a point P_S lying on the upper boundary. Without loss of generality, we consider the point P_S to be far from the point 0 and the depth of the liquid to be large. Then the value of u_r is calculated as a sum in formulas (13). A graph of the dependence of the terms of the sum on the ordinal number is presented in Fig. 1b. The theoretical value of u_r is zero. As is seen from Fig. 1b, paired terms $\partial\varphi_{i-k}/\partial r$ and $\partial\varphi_{i+k}/\partial r$ are present in the sum, these terms are approximately equal in modulus and opposite in sign. As the parameter l decreases, the moduli of the corresponding paired terms increase. Therefore, if $\partial\varphi_i/\partial r$ is calculated with a certain fixed accuracy, then the accuracy of calculation of u_r decreases. It follows that, assigning a certain accuracy of calculation of u_r, we obtain a limit for $|\partial\varphi_i/\partial r|$ and, consequently, for the minimum permissible value of the parameter l.

All the aforesaid also refers to calculation of u_z.

The described special features of calculation of the derivatives of the potential are explained by the complex behavior of the integrands arising in this problem.

Special Features of the Method of Integration over the Polar Angle. The dependence on the integration variable φ enters only the functions

$$Y_\beta^\alpha(c_1, c_2, \varphi) = \frac{\cos^\alpha \varphi}{(c_1 - c_2 \cos\varphi)^{\beta/2}}, \tag{21}$$

where $\alpha = 0, 1, 2$; $\beta = 3, 5$; $c_1, c_2 \in (0, \infty)$. To these functions the following integrals correspond:

$$I_\beta^\alpha(c_1, c_2) = \int_0^\pi Y_\beta^\alpha(c_1, c_2, \varphi)\, d\varphi. \tag{22}$$

At $c_1 = c_2$ the functions $Y_\beta^\alpha(c_1, c_2, \varphi)$ have a second-order discontinuity at the point $\varphi = 0$. In this case, the integral $I_\beta^\alpha(c_1, c_2 = c_1)$ diverges logarithmically. The equality $c_1 = c_2$ corresponds to the case where the points $M(r, z)$ and $P_{S_i}(\varphi, \varphi, h)$ coincide. This situation arises on each link of the broken line in calculation of the potential of the double layer. Here the double integral converges on each link of the broken line, i.e., in summation of integrals (22) the logarithmic features in formulas (11), (13), and (14) are cancelled out. The problem of choosing a

numerical method of integration which could be capable of eliminating the above-described difficulty arises here. The method of averages, where in contrast to the method of trapezoids and the Simpson method this special case does not arise, turned out to be most suitable.

Preliminary calculations showed that calculation of integrals (22) at each step of the algorithm leads to inadmissibly large expenditures of computer time. Thereafter, for calculation of integrals over the angle, the method of tabulation of functions, which allowed one to increase considerably the speed of calculations, was used.

Method of Tabulation of Functions. Using the method of averages as a guide, we consider the case where $c_1 \neq c_2$. Since the functions $Y_\beta^\alpha(c_1, c_2, \varphi)$ are bounded on the region of variation of φ, $\varphi \in [0, \pi]$, we pass over to their analogs normalized to unity at the point $\varphi = 0$:

$$Y_\beta^\alpha(c_1, c_2, \varphi) = \frac{Y_\beta^{'\alpha}(c_1/c_2, \varphi)}{c_2^{\beta/2}(c_1/c_2 - 1)^{\beta/2}}, \tag{23}$$

$$Y_\beta^{'\alpha}(c_1/c_2, \varphi) = \frac{(c_1/c_2 - 1)^{\beta/2}\cos^\alpha\varphi}{(c_1/c_2 - \cos\varphi)^{\beta/2}}. \tag{24}$$

We introduce the parameter $a = c_1/c_2$, $a \in (1, \infty)$. Then expression (24) is written as

$$Y_\beta^{'\alpha}(a, \varphi) = \frac{(a-1)^{\beta/2}\cos^\alpha\varphi}{(a - \cos\varphi)^{\beta/2}}. \tag{25}$$

The integral $I_\beta^\alpha(c_1, c_2)$ is transformed to

$$I_\beta^\alpha(c_1, c_2) = \frac{I_\beta^{'\alpha}(c_1/c_2)}{c_2^{\beta/2}(c_1/c_2 - 1)^{\beta/2}}, \tag{26}$$

$$I_\beta^{'\alpha}(a) = \int_0^\pi Y_\beta^{'\alpha}(a, \varphi)\, d\varphi. \tag{27}$$

The integral $I_\beta^{'\alpha}(a)$ is a function of one parameter a, which is determined by the mutual position of the points $M(r, z)$ and $P_{S_i}(\rho, \varphi, h)$. As is seen from (25), as the parameter a tends to infinity, the integral $I_\beta^{'\alpha}(a)$ asymptotically tends to its maximum value. Depending on the indices α and β it is equal to

$$\max_a I_\beta^{'\alpha}(a) = \begin{cases} \pi, & \text{if } \beta = 3,\ \alpha = 0, \\ 0, & \text{if } \beta = 3,\ \alpha = 1, \\ \pi, & \text{if } \beta = 5,\ \alpha = 0, \\ 0, & \text{if } \beta = 5,\ \alpha = 1, \\ \pi/2, & \text{if } \beta = 2,\ \alpha = 2. \end{cases} \tag{28}$$

Having prescribed a certain accuracy, we succeed in approximating the function $I_\beta^{'\alpha}(a)$ by a continuous curve. Then we can construct a table of values of the function at the points of division of the region of variation of the parameter a, which are the a coordinates of the nodes of the broken line. By an additional program we can construct tables for use of these values in the solution of the main problem. These tables are universal and do not depend on the initial parameters of the problem. Thus, in order not to calculate values of the function $I_\beta^{'\alpha}(a)$ in solving the problem and to increase the speed of solution, the tabulated functions were used in the work.

Special Features of the Method of Tabulation. We consider, as an example, the graph of the function $Y_3^{'0}(a, \varphi)$ of the variable φ at different values of the parameter a. As is seen from Fig. 2, for values of the parameter close to unity the function decreases rapidly near the zero point. Since the values of the parameter a that are actually

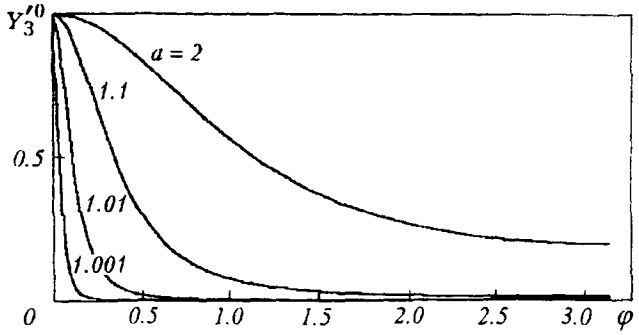

Fig. 2. Graph of the function $Y_3'^0(a, \varphi)$ for different values of the parameter a. φ, rad.

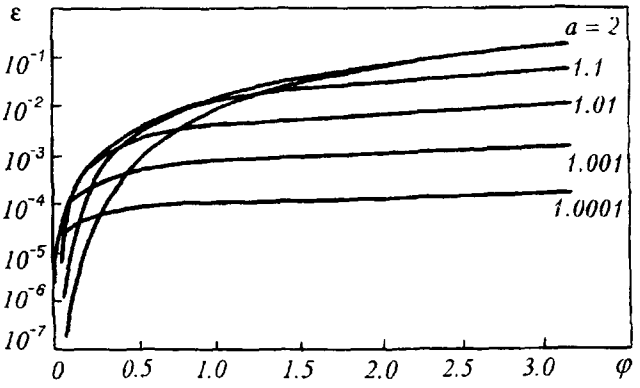

Fig. 3. Graph of the dependence of the approximation error for different values of the parameter a.

encountered in the solution of the problem reach a value of $1 + 10^{-10}$ in their minimum, none of the existing standard numerical methods of integration allows one to calculate an integral of this function with sufficient accuracy. To calculate integrals of the function $Y_\beta'^\alpha(a, \varphi)$ we developed a special numerical-analytical method. Therefore, they are calculated approximately, so that the integrals coincide within a prescribed accuracy with the initial ones on the interval $(0, \varphi^*)$. As a result of approximation of the integrands we found the following functions:

$$Y_\beta^*(a, \varphi) = \frac{(a-1)^{\beta/2}}{\left(a - 1 + \frac{\varphi^2}{2}\right)^{\beta/2}}, \tag{29}$$

$$Y_\beta'^\alpha(a, \varphi) \approx Y_\beta^*(a, \varphi). \tag{30}$$

For any α and β these functions satisfy our requirements, and integrals of them are calculated analytically:

$$I_3^*(a, \varphi^*) = \int_0^{\varphi^*} Y_3^*(a, \varphi)\, d\varphi = \frac{\sqrt{2}\,\varphi^*\sqrt{a-1}}{\sqrt{2(a-1) + (\varphi^*)^2}}, \tag{31}$$

$$I_5^*(a, \varphi^*) = \int_0^{\varphi^*} Y_5^*(a, \varphi)\, d\varphi = \frac{2}{3}(3(a-1) + (\varphi^*)^2)\frac{\sqrt{2}\,\varphi^*\sqrt{a-1}}{(2(a-1) + (\varphi^*)^2)^{3/2}}. \tag{32}$$

To integrate the initial functions on the remaining interval (φ^*, π) we used the method of averages for numerical integration.

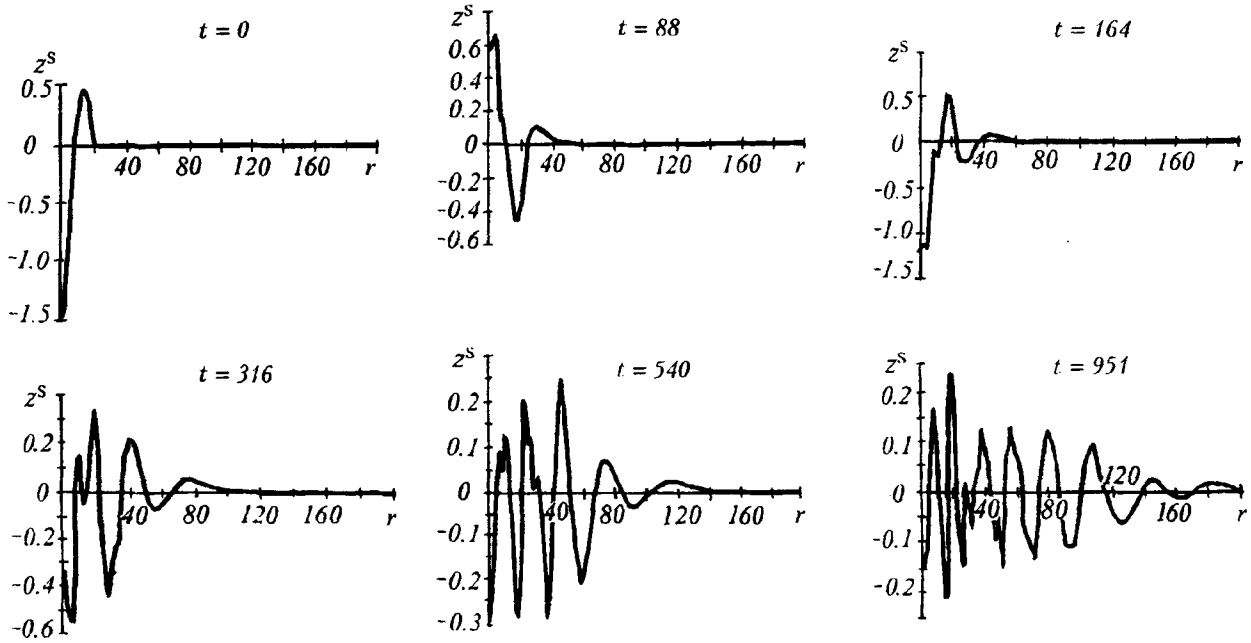

Fig. 4. Results of calculation of wave-structure formation in the fall of a cosmic object into the ocean. The values of the wave profiles at the indicated instants of time are given in kilometers. t, sec; z^S, r, km.

The parameter of division φ^* depends on the number of the approximating function β, the required accuracy of approximation ε, and the parameter a, i.e., $\varphi^*_\beta = \varphi^*_\beta(a, \varepsilon)$. We consider an algorithm for determination of φ^* on the example of the function $Y^0_\beta(a, \varphi)$. Figure 3 presents the dependence of the approximation error for different values of the parameter a. Using the envelope of graphs constructed for different values of the parameter a, having prescribed a certain accuracy, we can obtain the minimum value of the parameter of division φ^*.

Results of Calculation and Their Discussion. Numerical calculation of the evolution of an initial disturbance in the ocean showed that an initial shortwave disturbance produced on water initiates a longwave disturbance. This leads to the fact that a formed train of waves resembles a tsunami more and more. Figure 4 presents results of calculation of wave-structure formation in the fall of a cosmic object into the ocean. The initial profile of the surface had a height of 0.5 km and a valley at the origin of coordinates equal to 1.5 km. The maximum point of the initial profile corresponded to r = 15 km. The ocean depth was 5 km, the length of the region of solution of the problem was 200 km. The grid contained 131 nodes on the wave profile.

An analysis of the graphs in Fig. 4 shows that even at the stage of formation of the wave structure from the initial disturbance there exist two regions: the region of shortwave disturbances and the region of longwave flow. Further evolution of the disturbance leads to disappearance of the shortwave disturbances and development of longwave ones. These data confirm the hypothesis that the fall of cosmic bodies into the ocean can lead to formation of tsunami.

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NOTATION

$\vec{u}$, velocity vector of liquid; $\varphi(r, z, t)$, potential of the velocity; t, time variable; r, radial spatial variable of the cylindrical system of coordinates; z, spatial variable; S, curve of the surface profile; S_u, part of the surface profile corresponding to the upper boundary; S_r, part of the surface profile corresponding to the right-hand boundary; S_b, part of the surface profile corresponding to the lower boundary; $\gamma(P_S)$, density of the potential of the double layer at points on the liquid boundary; P_S, points of the boundary of the liquid volume; N, number of links in the approximating broken line; S_i, link of the broken line; i, index of the number of the link of the broken line;

Ω, region of integration; Ω_i, annular segment of the integration region; $M(r, z)$, $P(\rho, h)$, arbitrary points inside the region; $\partial/\partial n_P$, derivative along the normal to the surface at the point P; r_{MP}, distance between the points M and P; $d\sigma_P$, differential of the region of solution; $\varphi^S(M)$, limiting values of the potential of the velocity from inside; $\partial/\partial t$, partial derivative with respect to the variable t; $\partial/\partial z$, partial derivative with respect to the variable z; $\partial/\partial r$, partial derivative with respect to the variable r; z^S, z coordinate of the wave profile; u_r, component of the velocity vector along the r axis; u_z, component of the velocity vector along the z axis; Λ_r, finite-difference operator of differentiation with respect to r; $d\vec{S}_i$, differential of the length of the curve of the liquid boundary; $d\varphi$, differential of the polar angle; $K(r, z, \rho, \varphi, h)$, kernel of the integral representation of the velocity potential (the notation introduced in (9)); $Y^{\alpha}_{\beta}(c_1, c_2, \varphi)$, notation introduced by formulas (10); $\alpha, \beta, n_1, n_2, c_1, c_2$, parameters introduced after (10); $F(\rho, h)$, some arbitrary function; T, actual parameter in formulas (15)-(17); $\rho(T)$, $h(T)$, auxiliary functions introduced in transformation of the formulas of calculation of the velocity potential and its derivatives to the form (15)-(17); ΔL^*, step of receding; l^*, adjustable parameter in formula (20); ΔL_{S_i}, length of the i-th link of the broken line; $I^{\alpha}_{\beta}(c_1, c_2)$, notation of the integral of the function $Y^{\alpha}_{\beta}(c_1, c_2, \varphi)$ with respect to the polar angle introduced by formula (22); $Y'^{\alpha}_{\beta}(c_1/c_2, \varphi)$, analog of the function $Y^{\alpha}_{\beta}(c_1, c_2, \varphi)$ normalized to unity at the point $\varphi = 0$; a, parameter introduced in (25) as the argument of the function $Y'^{\alpha}_{\beta}(a, \varphi)$; $I'^{\alpha}_{\beta}(a)$, notation of the integral of the function $Y'^{\alpha}_{\beta}(a, \varphi)$ with respect to the polar angle introduced by formula (27); $(0, \varphi^*)$, interval of application of the approximating relation in calculation of $I'^{\alpha}_{\beta}(a)$; φ^*, parameter of division of the interval of integration; $Y^*_{\beta}(a, \varphi)$, approximating relation for the function $Y'^{\alpha}_{\beta}(a, \varphi)$; ε, required accuracy of approximation.

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